1. (10 points) Determine whether or not the following statements are true or false. If you think a statement is true, give your reasoning. If you think it is false, provide a counterexample.

(a) For all sets A and B, if B ⊆ ̄A, then A ∩ B = ∅.

TRUE if B exists in the set of not A, then the intersection of A and B would be a empty subset

(b) For all sets A, B, C, if B ⊆ C and A ∩ C = ∅, then A ∩ B = ∅.

TRUE because if b exists inside C and A and C have no subset, then B and A would also have no subset because C contains all elements in B.

2. (10 points) Indicate which of the following relationships are true and which are false, together with a brief explanation in words why you think that is the case:

(a) Z+ ⊆ Q.

TRUE The set Z contains all the positive integers and the set Q contains set of all rational numbers. So every element in the set Z is the element of the set Q.

(b) Q ⊆ Z.

FALSE For q = 1, the set of rational numbers is same as the set of integers. So the relationship is false, because ½ ∈ Q but ½ does not exist in Z

(c) Q ∩ R = Q.

TRUE The set Q contains set of all rational numbers and the set R contains set of all real numbers. All real numbers include the rational set of numbers.

(d) Z+ ∩ R = Z+.

TRUE The set Z contains all the positive integers and the set R contains set of all real numbers. All real numbers include the sets of positive integers.

(e) ∅ ⊂ N.

TRUE ∅ represents the empty set, which is a subset of every set, including the set of natural numbers N.

3. (10 points) Prove the distributive law for any three sets A, B, C: A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C). You can use any method of proof. For example, for a formal logic proof you might want to consider an element x ∈ A ∪ (B ∩ C) and construct a chain of logical deductions to show that x also belongs to (A ∪ B) ∩ (A ∪ C). Or you could use Venn diagrams.

⟷x ∈ A or x ∈ B and x ∈ C

⟷x ∈ A or x ∈ B and x ∈ A or x ∈ C

⟷x ∈ A ∪ B and x ∈ A ∪ C

x ∈ (A ∪ B) ∩ (A ∪ C)

4. (10 points) Let A = {a, b, c, d} and B = {y, z}. Determine A × B and B × A. Are they equal?

A x B = {(a,y), (a,z), (b,y), (b,z), (c,y), (c,z), (d,y), (d,z)}

B x A = {(y,a), (y,b), (y,c), (y,d), (z,a), (z,b), (z,c), (z,d)}

No they are not equal

5. (10 points) Prove or disprove that for all sets A, B and C, we have (i) A × (B − C) = (A × B) − (A × C).

This relationship will work.

Let (x,y) ∈ A x (B-C)

If and only if x ∈ A and y ∈ B-C

If and only if x ∈ A and y ∈ B but x ∈ A and y not ∈ C

If and only if (x,y) ∈ AxB - (AxC)

So (x,y) ∈ A x (B-C) ↔ (x,y) ∈ A x B - (A x C)

(if a ∈ X implies a ∈ Y then X Ｃ Y)

(ii) Ā × (B ∪ C) = A × (B ∪ C).

This relationship does not work.

Consider: U{1,2,3,4,5} A{2,3,4,5} B{1,2,4} C{1,2,5}

Then the union of B and C = {1,2,4,5}

x A = {(2,1),(2,2),(2,4),(2,5), (3,1), etc.}

Then the complements are -A= {1} -B= {3,5} -C= {3,4}

A × (B ∪ C) = {1,1),(1,2),(1,3),(1,4),(1,5),(2,3),(3,3),(4,3),(5,3)} which is not = to the original

So the left and right do not equal each other

6. (10 points) Let E be the set of even integers and O be the set of odd

integers. Define a function: f : E × O → Z such that f (x, y) = xy. Is f one-to-one? Is f onto? For either question, if your answer is yes, then prove it; if not, then provide a counterexample.

One-to-one:

f(0,1) = 0x1 = 0 f(0,3) = 0x3 = 3

f(0,1) = f(0,3) but (0,1) does not = (0,3) so it is NOT one-to-one

Onto:

Since x∈ E there exists an int r where x = 2r f(x,y) = 1 so xy = 1

Therefore, 2ry = 1. However, this is a contradiction because the left side is even and the right is odd so f is NOT onto

7. (10 points) Let f : A → B and g : B → C be functions. Let h : A → C be their composition, i.e., h(a) = g(f (a)).

(a) Prove that if f and g are surjections, then so is h.

Since f is a surjection, for every element b in the codomain B, there exists an element a in the domain A such that f(a) = b

Similarly, since g is a surjection, for every element c in the codomain C, there exists an element b in the domain B such that g(b) = c

Therefore, h(a) = g(f(a)) = g(b) = c. so h is a surjection

(b) Prove that if f and g are b ijections then so is h.

Since f is an injection, f(a1) and f(a2) are distinct elements in the domain B.

Similarly, since g is an injection, g(f(a1)) and g(f(a2)) are distinct elements in the domain C.

Since they are distinct, then h(a1) and h(a2) are also distinct making h an injection.

8. (10 points) Determine if the following are functions. The domain is R and the co-domain is R. (i)f (x) = 1/x,

If x = 0 then 1/0 which is undefined. Domain and range of the function is R - {0}

(ii)f (x) = √x,

Negative numbers do not work because they would make an imaginary number. Domain and range of the function is [0,∞)

(iii)f (x) = ±√x2 + 1.

It is not a well-defined function because it does not pass the vertical line test. The issue here is the use of the "±" symbol. The function can have two possible outputs: one positive and one negative value.

9. (20 points) Determine if these functions from Z (domain) to Z (co-domain) are one-to-one (injective).

(i)f (n) = n − 1,

the function f(n) = n - 1 is one-to-one. we have two different inputs n1 and n2 such that f(n1) = f(n2). n1=1,n2=2: f(1) = 0 f(2)= 1 for n1 and n2 we get different values so the function is one-to-one.

(ii)f (n) = n2 + 1,

The function f(n) = n2 + 1 is not one-to-one. For n1= -1,n2= 1 f(n1) = 1^2 + 1 = 2

f(n2) = (-1)^2 + 1 = 2 Since f(n1) = f(n2),

(iii)f (n) = n3,

f(n) = n^3 is one-to-one

For n = 1, f(1) = 1^3 = 1

For n = 2, f(2) = 2^3 = 8

For n = -2, f(-2) = (-2)^3 = -8

For n = 0, f(0) = 0^3 = 0

(iv)f (n) = ⌈n/2⌉.

f(n) = ⌈n/2⌉ is one-to-one

For n = 1, f(1) = ⌈1/2⌉ = 1

For n = 2, f(2) = ⌈2/2⌉ = 1

For n = 3, f(3) = ⌈3/2⌉ = 2

For n = 4, f(4) = ⌈4/2⌉ = 2

10. (20 points) Determine if these functions from Z × Z (domain) to Z (co-domain) are onto (surjective).

(i)f (m, n) = m + n,

f(m,n)=m+n is onto

Every integer in the range of the function has at least one pre-image in the domain.

(ii)f (m, n) = m2 − n2,

f(m,n)=m^2-n^2 is not onto

The function f(m,n) = m^2 - n^2 is not onto because there is no solution for z = -1, since m^2 and n^2 are always non-negative.

(iii)f (m, n) = m,

f(m,n)=m is not onto

f(m,n) = m is not onto because it only takes values from Z, a subset of Z x Z, but not all values in Z x Z can be expressed as m. f(m,n) = m.

(iv)f (m, n) = |m| − |n|.

f(m,n)=|m|−|n| is onto Every integer in the range of the function has at least one pre-image in the domain.

11. (10 points) Find f ◦ g and g ◦ f if f (x) = x2 + 1 and g(x) = x + 2 if the

functions are from R (domain) to R (co-domain).

f o g = f [ g [x] ]

= f [ x+2 ] = ( x+2 )^2 + 1 ]

= [ (x+2)^2 +1 ]

= x^2 + 4x + 4 +1 = x^2 + 4x + 5]

f o g = x^2 + 4x + 5

g o f = g [ f (x) ]

= g ( x^2 + 1 )

= ( x^2 + 1 ) + 2

= x^2 + 1 + 2

g o f = x^2 + 3